Boolean Functions

$(B, +, *, ')$ → Boolean Alg.

0, 1 → specific elements of B.

variable → an arbitrary element of B.

- Boolean Fn. or Boolean Polynomial is an expression which is derived from a finite number of applications of the operations $+$, $*$ and $'$ to the elements of B.

e.g. $a*b$ or ab (we omit $*$), $a'+b'$, $a'+b$ are Boolean fns.

- In Boolean Alg., we have →

$a+a=a$ by idempotent law

or $2a=a$

$3a = a+a+a = 2a+a = a+a = a$

Sly, $na=a$ ∀ +ve int. 'n'.

$a^2 = a*a = a.a = a$

$a^3 = a^2 a = a.a = a$

Sly, $a^k = a$ ∀ +ve int. 'k'.

∴ no multiples or powers appear in the Boolean polynomials.

Minimal Polynomial (minterm)

A Boolean expression of 'n' variables $x_1, x_2, .., x_n$ is said to be a minterm or a minimal poly. if it is of the form

$f_1(x_1)\, f_2(x_2) \,..\, f_n(x_n)$ → Product

where $f_i(x_i) = x_i$ or x_i' ∀ $i = 1, 2, .., n$

e.g. $x_1 \cdot x_2$, $x_1 x_2'$, $x_1' x_2$, $x_1' x_2'$

Th. There are exactly 2^n minterms in n variables $x_1, x_2, .., x_n$.

Pf. A minterm in 'n' variables is —

$f_1(x_1) f_2(x_2) \ldots f_n(x_n)$

where $f_i(x_i) = x_i$ or x_i' $\forall i = 1, 2 .., n$

There are two ways of selecting $f_i(x_i)$ viz x_i or x_i' $\forall i = 1, 2, , n$.

$\therefore$ there are 2^n different minterms in n variables.

Disjunctive Normal Form (DNF)

A Boolean fn. in n variables $x_1, x_2, \ldots x_n$ is said to be in DNF if it is sum of minterms. 1 & 0 are in DNF.

$\therefore$ in DNF, a fn. is a sum of the terms of the type

$f_1(x_1) f_2(x_2) \ldots, f_n(x_n)$

where $f_i(x_i) = x_i$ or x_i' $\cdot \forall i = 1, 2 . , n$.

and no two terms are same.

It is sum of products

Ex. To write the fn $f = (xy' + xz)' + x'$ in DNF

Soln.
$$f = (xy' + xz)' + x'$$
$$= (xy')' (xz)' + x' ; \text{ De-morgan law}$$
$$= (x' + y)(x' + z') + x', \quad "$$

$= x' + yz' + x'$; distr law

$= x' + x' + yz'$; comm.

$= x' + yz'$; idempotent

$$= \underbrace{x'.(y+y').(z+z')}_{\text{we introduce terms of } y \,\&\, z \text{ as } y+y'=1 \,\&\, z+z'=1} + \underbrace{(x+x')}_{\text{introduce term of } x \text{ as } x+x'=1} yz'$$

$= x'yz + \underline{x'yz'} + x'y'z + x'y'z' + xyz' + \underline{x'yz'}$

$= \underbrace{x'yz}_{\text{Product}} + x'yz' + x'y'z + x'y'z' + xyz'$

$\underbrace{\qquad\qquad}_{\text{Sum}}$ → DNF → Sum of Products

Complete DNF

A DNF in 'n' variables which contains all the 2^n min terms is called Complete DNF e.g.

(i) $f = xy + x'y + xy' + x'y'$

is a complete DNF in 2 vars x & y.

(ii) $f = xyz + xy'z + xyz' + x'yz + x'yz' + x'y'z + xy'z' + x'y'z'$

Exm To find & simplify the function specified by table— (1a)

Row	x	y	z	$f(x,y,z)$
1	1	1	1	0
2	1	1	0	$\boxed{1}$
3	1	0	1	0
4	1	0	0	$\boxed{1}$
5	0	1	1	0
6	0	1	0	0
7	0	0	1	0
8	0	0	0	0

Identify 1's in colm of f.
2nd row & 4th row.

2nd row → expression : $x\,y\,z'$
$\quad\downarrow\ \downarrow\ \downarrow$
$\quad 1\ \ 1\ \ 0'=1 = 1.1.1=1$

4th row → expression : $x\,y'\,z'$
$\quad\downarrow\ \downarrow\ \downarrow$
$\quad 1\ \ 0'=1\ \ 0'=1 = 1.1.1=1$

$\therefore f = xyz' + xy'z'$

Now we simplify 'f' as—

$f = xyz' + xy'z'$
$= x(y+y')\cdot z' = x.1.z' = xz'$

Ans